# Learning from Corrupted Binary Labels via Class-Probability Estimation

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# Learning from noisy labels







### Learning from positive and unlabelled data





Goal: good classification wrt distribution D

#### Learning from corrupted labels



Goal: good classification wrt (unobserved) distribution D

# Learning from corrupted labels: applications

#### Learning from noisy annotators



# Implicit feedback recommendation









#### Habitat modelling



### Talk summary

#### Can we learn a good classifier from corrupted samples?

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Yes, if we make assumptions on:

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Yes, if we make assumptions on:

- the corruption process
- (optionally) the true distribution

### Solution strategy

What we do:

- write down the distribution we want to observe samples from
- compare to distribution we actually observe samples from
- 3 agree upon measure of performance
- Igure out how to correct for discrepancy between (1) and (2)

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### Solution sneak peek

What we suggest:

- treat corrupted labels as if they were uncorrupted
- train class-probability estimator (e.g. logistic regression)
- threshold predictions appropriately



#### Comment: why not be unhinged?

Precursor to unhinged learning work for label noise

Here, we consider a broader class of corruptions

• some results similar in spirit to "noise immunity"







Binary classification and class-probability estimation

#### Learning from binary labels: distributions

Fix instance space  $\mathcal{X}$  (e.g.  $\mathbb{R}^N$ )

Underlying distribution D over  $\mathfrak{X} \times \{\pm 1\}$ 

Constituent components of D:

$$(\mathbf{P}(x), \mathbf{Q}(x), \pi) = (\mathbb{P}[\mathsf{X} = x | \mathsf{Y} = 1], \mathbb{P}[\mathsf{X} = x | \mathsf{Y} = -1], \mathbb{P}[\mathsf{Y} = 1])$$

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#### Learning from binary labels: example



### Class-probability estimation

**Classification**: estimate sign $(\eta(x) - \frac{1}{2})$ 

- Bayes-optimal decision boundary
- returned by e.g. SVM with universal kernel

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#### **Class-probability estimation**: estimate $\phi \circ \eta$ for invertible $\phi$

- e.g. logistic regression:  $\phi: z \mapsto \frac{1}{1+e^{-z}}$
- e.g. AdaBoost:  $\phi: z \mapsto \frac{1}{1+e^{-2z}}$

#### Class-probability estimation useful when going beyond 0-1 error

#### Classification performance measures

General classification performance measure expressible as (Narasimhan et al., 2014):

 $\Psi(\text{FNR}^D(f), \text{FPR}^D(f), \pi)$ 

where

$$\operatorname{FNR}^{D}(f) = \mathbb{P}_{\mathsf{X} \sim P}(f(\mathsf{X}) = -1)$$
  
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Examples:

- 0-1 error  $\rightarrow \Psi$ :  $(u, v, p) \rightarrow p \cdot u + (1-p) \cdot v$
- Balanced error  $\rightarrow \Psi : (u, v, p) \rightarrow (u + v)/2$

• F-score 
$$\rightarrow \Psi$$
:  $(u, v, p) \rightarrow \frac{2 \cdot p \cdot (1-u)}{p + p \cdot (1-u) + (1-p) \cdot v}$ 

#### Class-probabilities and classification

Most "reasonable" performance measures  $\Psi$  optimised by

 $f^*: x \mapsto \operatorname{sign}(\eta(x) - t)$ 

- 0-1 error  $\rightarrow t = \frac{1}{2}$
- Balanced error  $\rightarrow t = \pi$
- F-score  $\rightarrow$  optimal *t* depends on *D* 
  - (Lipton et al., 2014, Koyejo et al., 2014)

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#### Can optimise such $\Psi$ using a class-probability estimator

### Optimising performance measures

Simple algorithm to optimise performance measure  $\Psi$ :

- compute class-probability estimates  $\hat{\eta}$  (e.g. by logistic regression)
- tune threshold  $\hat{t}$  to optimise  $\Psi$  on validation set
- return classifier

$$\hat{f}: x \mapsto \operatorname{sign}(\hat{\eta}(x) - \hat{t})$$

Resulting classifier  $\hat{f}$  is consistent (Narasimhan et al., 2014)

surrogate regret bounds also exist (Kotlowski & Dembczynski, 2015)

# Assumed corruption model



Samples from clean distribution  $D = (P, Q, \pi)$ 

Goal: good classification wrt distribution D

Learning from corrupted binary labels

Samples from corrupted distribution  $\overline{D} = (\overline{P}, \overline{Q}, \overline{\pi})$ 

Goal: good classification wrt (unobserved) distribution D

Learning from corrupted binary labels

Samples from corrupted distribution  $\overline{D} = (\overline{P}, \overline{Q}, \overline{\pi})$ , where

$$\bar{P} = (1 - \alpha) \cdot P + \alpha \cdot Q$$
$$\bar{Q} = \beta \cdot P + (1 - \beta) \cdot Q$$

and  $\bar{\pi}$  is arbitrary

- $\alpha, \beta$  are noise rates
- mutually contaminated distributions (Scott et al., 2013)

Goal: good classification wrt (unobserved) distribution D

### Special case: label noise

Labels flipped with probability  $\rho_+, \rho_ \bar{\pi} = (1 - \rho_+ - \rho_-) \cdot \pi + \rho_+ + \rho_ \alpha = \bar{\pi}^{-1} \cdot (1 - \pi) \cdot \rho_ \beta = (1 - \bar{\pi})^{-1} \cdot \pi \cdot \rho_+$ 



### Special case: PU learning

Observe M instead of Q

 $\bar{\pi} = arbitrary$ 

- $\bar{P} = 1 \cdot P + 0 \cdot Q$
- $$\begin{split} \bar{Q} &= M \\ &= \pi \cdot P + (1 \pi) \cdot Q \end{split}$$



# Caution: two faces of PU learning



Can also cast PU learning as specific case of asymmetric label noise (Elkan and Noto, 2008)

- +'ves flipped with censoring probability c
- -'ves flipped with probability 0

"Case-controlled" versus "censoring" versions of the problem

#### Corrupted class-probabilities

Structure of corrupted class-probabilities underpins analysis
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Structure of corrupted class-probabilities underpins analysis

Proposition For any  $D, \overline{D}$ ,

$$\bar{\boldsymbol{\eta}}(\boldsymbol{x}) = \boldsymbol{\phi}_{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\pi}}(\boldsymbol{\eta}(\boldsymbol{x}))$$

where  $\phi_{\alpha,\beta,\pi}$  is strictly monotone for fixed  $\alpha,\beta,\pi$ .

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Follows from Bayes' rule:

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# Corrupted class-probabilities: special cases

Label noise

**PU learning** 

$$\bar{\boldsymbol{\eta}}(x) = (1 - \boldsymbol{\rho}_{+} - \boldsymbol{\rho}_{-}) \cdot \boldsymbol{\eta}(x) + \boldsymbol{\rho}_{-}$$

$$\rho_+, \rho_-$$
 unknown

(Natarajan et al., 2013)

 $\pi$  unknown

(Ward et al., 2009)

 $\bar{\eta}(x) = \frac{\pi \cdot \eta(x)}{\pi \cdot \eta(x) + (1-\pi) \cdot \bar{\pi}}$ 

# Corrupted class-probabilities: comments

Form of  $\bar{\eta}$  implies suitable choice of function class

e.g. if  $\eta : x \mapsto \frac{1}{1+e^{-s(x)}}$ , then neural network is well-specified for  $\bar{\eta}$ • if you can't be unhinged, be neurotic

Label noise

PU learning

$$\bar{\eta}(x) = a \cdot \frac{1}{1 + e^{-s(x)}} + b$$

$$\bar{\eta}(x) = \frac{1}{a+b\cdot e^{-s(x)}}$$

# Roadmap



Kernel logistic regression



Exploit monotone relationship between  $\eta$  and  $\bar{\eta}$ 



Kernel logistic regression

# Classification with noise rates

Recap: class-probabilities and classification

Most "reasonable" performance measures optimised by

 $f^*: x \mapsto \operatorname{sign}(\eta(x) - t)$ 

- 0-1 error  $\rightarrow t = \frac{1}{2}$
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- F-score  $\rightarrow$  optimal *t* depends on *D* 
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## We can relate this to thresholding of $\bar{\eta}$ !

# Corrupted class-probabilities and classification

By monotone relationship,

$$\eta(x) > t \iff \bar{\eta}(x) > \phi_{\alpha,\beta,\pi}(t).$$

## Threshold $\bar{\eta}$ at $\phi_{\alpha,\beta,\pi}(t) \rightarrow$ optimal classification on D

# Optimal classifiers for 0-1 error: special cases

## Label noise

### PU learning

$$\operatorname{sign}\left(\bar{\eta}(x) - \frac{1 - \rho_+ + \rho_-}{2}\right) \qquad \qquad \operatorname{sign}\left(\bar{\eta}(x) - \frac{\bar{\pi}}{\bar{\pi} + 2 \cdot (1 - \bar{\pi}) \cdot \pi}\right)$$

## Thresholding at $\frac{1}{2}$ is in general not optimal

- using standard binary classifier will fail
- but changing the threshold overcomes this

# Optimising performance measures from corrupted samples

Simple algorithm to optimise performance measure  $\Psi$ :

- compute corrupted class-probability estimates  $\hat{\eta}$  (e.g. by logistic regression)
- tune threshold  $\hat{t}$  to optimise  $\Psi$  on validation set
- return classifier

$$\hat{f}: x \mapsto \operatorname{sign}(\hat{\bar{\eta}}(x) - \hat{t})$$

Can derive surrogate regret bounds as before

Classification scheme requires:

η

t

•  $\alpha, \beta, \pi$ 



Classification scheme requires:

•  $ar\eta o$  class-probability estimation

t

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Kernel logistic regression

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Kernel logistic regression

Classification scheme requires:

- $ar\eta o$  class-probability estimation
- t 
  ightarrow constant, or using  $\bar{\Psi}$
- $\alpha, \beta, \pi \rightarrow$  can we estimate these?



Kernel logistic regression

# Estimating noise rates: some bad news

 $\pi$  strongly non-identifiable!

•  $\bar{\pi}$  allowed to be arbitrary (e.g. PU learning)

 $\alpha,\beta$  non-identifiable without assumptions (Scott et al., 2013)

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Can we estimate  $\alpha, \beta$  under assumptions?

# Weak separability assumption

Assume that *D* is "weakly separable":

$$\min_{x \in \mathcal{X}} \eta(x) = 0$$
$$\max_{x \in \mathcal{X}} \eta(x) = 1$$

- i.e. ∃ deterministically +'ve and -'ve instances
- weaker than full separability

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## Assumed range of $\eta$ constrains observed range of $\bar{\eta}$ !

# Estimating noise rates

## Proposition

Pick any weakly separable D. Then, for any  $\overline{D}$ ,

$$\alpha = \frac{\eta_{\min} \cdot (\eta_{\max} - \bar{\pi})}{\bar{\pi} \cdot (\eta_{\max} - \eta_{\min})} \text{ and } \beta = \frac{(1 - \eta_{\max}) \cdot (\bar{\pi} - \eta_{\min})}{(1 - \bar{\pi}) \cdot (\eta_{\max} - \eta_{\min})}$$

where

$$\eta_{\min} = \min_{x \in \mathcal{X}} \bar{\eta}(x)$$
$$\eta_{\max} = \max_{x \in \mathcal{Y}} \bar{\eta}(x)$$

### lpha,eta can be estimated from corrupted data alone

# Estimating noise rates: special cases

Label noise

$$egin{aligned} & oldsymbol{
ho}_+ = 1 - \eta_{ ext{max}} \ & oldsymbol{
ho}_- = \eta_{ ext{min}} \ & \pi = rac{ar{\pi} - \eta_{ ext{min}}}{\eta_{ ext{max}} - \eta_{ ext{min}}} \end{aligned}$$

**PU learning** 

$$egin{aligned} lpha &= 0 \ eta &= \pi \ &= rac{1 - \eta_{ ext{max}}}{\eta_{ ext{max}}} \cdot rac{ar{\pi}}{1 - ar{\pi}} \end{aligned}$$

(Elkan and Noto, 2008), (Liu and Tao, 2014)

In these cases,  $\pi$  can be estimated as well

# Estimating noise rates: comments

Given estimates  $\hat{\eta}$ , can use plugin versions of  $\eta_{\min}, \eta_{\max}$ 

Estimating order statistics not ideal

- estimates of e.g.  $\pi$  will be sensitive to errors in  $\eta_{\min}, \eta_{\max}$
- under stronger assumptions on D, more well-behaved estimators possible, e.g.

$$\mathbf{\rho}_{+} = \mathop{\mathbb{E}}_{\mathsf{X} \sim P} [\boldsymbol{\eta}(\mathsf{X})]$$

## Optimal classification in general requires $\alpha, \beta, \pi$



Kernel logistic regression

Optimal classification in general requires  $\alpha, \beta, \pi$ 

• when does  $\phi_{\alpha,\beta,\pi}(t)$  not depend on  $\alpha,\beta,\pi$ ?





# Classification without noise rates

# Balanced error (BER) of classifier

Balanced error (BER) of a classifier  $f: \mathcal{X} \to \{\pm 1\}$  is:

$$\mathrm{BER}^D(f) = \frac{\mathrm{FPR}^D(f) + \mathrm{FNR}^D(f)}{2}$$

for false positive and negative rates  $FPR^{D}(f)$ ,  $FNR^{D}(f)$ 

- average classification performance on each class
- favoured when classes are imbalanced

BER "immunity" under corruption Proposition (c.f. (Zhang and Lee, 2008)) For any  $D, \overline{D}$ , and any classifier  $f: \mathcal{X} \to \{\pm 1\}$ ,  $BER^{\overline{D}}(f) = (1 - \alpha - \beta) \cdot BER^{D}(f) + \frac{\alpha + \beta}{2}$  BER "immunity" under corruption Proposition (c.f. (Zhang and Lee, 2008)) For any  $D, \overline{D}$ , and any classifier  $f: \mathfrak{X} \to \{\pm 1\}$ ,

$$\operatorname{BER}^{\overline{D}}(f) = (1 - \alpha - \beta) \cdot \operatorname{BER}^{D}(f) + \frac{\alpha + \beta}{2}$$

#### Minimising corrupted BER minimises clean BER!

can ignore corruption process

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Trivially, we also have

regret<sub>BER</sub><sup>*D*</sup>
$$(f) = (1 - \alpha - \beta)^{-1} \cdot \text{regret}_{BER}^{\overline{D}}(f).$$

i.e. good corrupted BER  $\implies$  good clean BER

# BER "immunity" & class-probability estimation

Can optimise corrupted BER via class-probability estimation:

- compute corrupted class-probability estimates  $\hat{\eta}$
- threshold  $\hat{ar{\eta}}$  around corrupted base rate  $ar{\pi}$

# BER "immunity" & class-probability estimation

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For strongly proper composite  $\ell$ , and scorer  $s: \mathfrak{X} \to \mathbb{R}$ ,

$$\operatorname{regret}_{\operatorname{BER}}^{\overline{D}}(f_s) \leq C_{\ell,\pi} \cdot \sqrt{\operatorname{regret}_{\ell}^{\overline{D}}(s)}.$$

i.e. can make  $\operatorname{regret}_{\operatorname{BER}}^{D}(f) \to 0$  by class-probability estimation

# BER "immunity" under corruption: proof

From (Scott et al., 2013),

$$\begin{bmatrix} \operatorname{FPR}^{\overline{D}}(f) & \operatorname{FNR}^{\overline{D}}(f) \end{bmatrix}^T = \begin{bmatrix} \operatorname{FPR}^{D}(f) & \operatorname{FNR}^{D}(f) \end{bmatrix}^T \cdot \begin{bmatrix} 1 - \beta & -\alpha \\ -\beta & 1 - \alpha \end{bmatrix} \\ + \begin{bmatrix} \beta & \alpha \end{bmatrix}^T,$$

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and  $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$  is an eigenvector of  $\begin{bmatrix} 1 - \beta & -\alpha \\ -\beta & 1 - \alpha \end{bmatrix}$ 

# BER "immunity" under corruption: comments

Results do not rely on weak separability assumption for *D* 

Regret relation does not rely on model being well-specified

• close to best corrupted BER in class  $\mathcal{H} \to$  close to best clean BER in class  $\mathcal{H}$
#### Corollary: AUC "immunity" under corruption

Area under ROC curve (AUC) of a scorer  $s \colon \mathcal{X} \to \mathbb{R}$ :

$$AUC^{D}(s) = \mathbb{E}_{\mathsf{X} \sim P, \mathsf{X}' \sim Q} \left[ \llbracket s(\mathsf{X}) > s(\mathsf{X}') \rrbracket + \frac{1}{2} \llbracket s(\mathsf{X}) = s(\mathsf{X}') \rrbracket \right]$$

probability of random +'ve scoring higher than random -'ve

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probability of random +'ve scoring higher than random -'ve

#### Corollary

For any  $D, \overline{D}$ , and scorer  $s \colon \mathcal{X} \to \mathbb{R}$ ,

$$\operatorname{AUC}^{\overline{D}}(s) = (1 - \alpha - \beta) \cdot \operatorname{AUC}^{D}(s) + \frac{\alpha + \beta}{2}$$

#### Pairwise ranking $\rightarrow$ can ignore corruption process

#### Are other measures "immune"?

BER is only (non-trivial) performance measure for which:

- corrupted risk = affine transform of clean risk
  - because of eigenvector interpretation
- corrupted threshold is independent of  $\alpha, \beta, \pi$ 
  - because of nature of  $\phi_{\alpha,\beta,\pi}$

Other performance measures ightarrow need (one of)  $lpha,eta,\pi$ 

# Experiments

#### Experimental setup

Injected label noise on UCI datasets

Estimate corrupted class-probabilities via neural network

• well-specified if *D* linearly separable:

$$\eta(x) = \sigma(\langle w, x \rangle) \implies \overline{\eta}(x) = a \cdot \sigma(\langle w, x \rangle) + b$$

Evaluate:

- BER performance on clean test set
  - corrupted data used for training and validation
- 0-1 performance on clean test set
- reliability of noise estimates

## Experimental results: BER immunity

Generally, low observed degradation in BER

Dataset	Noise	1 - AUC (%)	<b>BER</b> (%)
segment	None	$\textbf{0.00} \pm \textbf{0.00}$	$0.00\pm0.00$
	$(\rho_+, \rho) = (0.1, 0.0)$	$0.00\pm0.00$	$0.01\pm0.00$
	$(\rho_+, \rho) = (0.1, 0.2)$	$\textbf{0.02} \pm \textbf{0.01}$	$\textbf{0.90} \pm \textbf{0.08}$
	$(\rho_+, \rho) = (0.2, 0.4)$	$0.03\pm0.01$	$\textbf{3.24} \pm \textbf{0.20}$
spambase	None	$\textbf{2.49} \pm \textbf{0.00}$	$\textbf{6.93} \pm \textbf{0.00}$
	$(\rho_+, \rho) = (0.1, 0.0)$	$\textbf{2.67} \pm \textbf{0.02}$	$\textbf{7.10} \pm \textbf{0.03}$
	$(\rho_+, \rho) = (0.1, 0.2)$	$\textbf{3.01} \pm \textbf{0.03}$	$\textbf{7.66} \pm \textbf{0.05}$
	$(\rho_+, \rho) = (0.2, 0.4)$	$\textbf{4.91} \pm \textbf{0.09}$	$10.52\pm0.13$
mnist	None	$\textbf{0.92} \pm \textbf{0.00}$	$\textbf{3.63} \pm \textbf{0.00}$
	$(\rho_+, \rho) = (0.1, 0.0)$	$\textbf{0.95} \pm \textbf{0.01}$	$\textbf{3.56} \pm \textbf{0.01}$
	$(\rho_+, \rho) = (0.1, 0.2)$	$\textbf{0.97} \pm \textbf{0.01}$	$\textbf{3.63} \pm \textbf{0.02}$
	$(\rho_+, \rho) = (0.2, 0.4)$	$1.17\pm0.02$	$\textbf{4.06} \pm \textbf{0.03}$

#### Experimental results: 0-1 error

0-1 error with estimated noise rates  $\sim$  using oracle noise rates

Dataset	Noise	$\text{ERR}_{\text{est}}(\%)$	$\text{ERR}_{\text{oracle}}(\%)$
segment	None	$\textbf{0.00} \pm \textbf{0.00}$	$\textbf{0.00} \pm \textbf{0.00}$
	$(\rho_+, \rho) = (0.1, 0.0)$	$0.01 \pm 0.00$	$0.01\pm0.00$
	$(\rho_+, \rho) = (0.1, 0.2)$	$0.31\pm0.05$	$\textbf{0.30} \pm \textbf{0.05}$
	$(\rho_+, \rho) = (0.2, 0.4)$	$0.31\pm0.06$	$\textbf{0.27} \pm \textbf{0.06}$
	None	$\textbf{6.52} \pm \textbf{0.00}$	$\textbf{6.52} \pm \textbf{0.00}$
spambase	$( ho_+, ho) = (0.1,0.0)$	$\textbf{6.88} \pm \textbf{0.03}$	$\textbf{6.89} \pm \textbf{0.03}$
	$(\rho_+, \rho) = (0.1, 0.2)$	$7.51 \pm 0.05$	$\textbf{7.48} \pm \textbf{0.05}$
	$(\rho_+, \rho) = (0.2, 0.4)$	$10.82\pm0.31$	$10.26\pm0.12$
	None	$\textbf{3.63} \pm \textbf{0.00}$	$\textbf{3.63} \pm \textbf{0.00}$
mnist	$( ho_+, ho) = (0.1,0.0)$	$\textbf{3.55} \pm \textbf{0.01}$	$\textbf{3.55} \pm \textbf{0.01}$
	$(\rho_+, \rho) = (0.1, 0.2)$	$\textbf{3.62} \pm \textbf{0.02}$	$\textbf{3.62} \pm \textbf{0.02}$
	$(\rho_+, \rho) = (0.2, 0.4)$	$\textbf{4.06} \pm \textbf{0.03}$	$\textbf{4.05} \pm \textbf{0.03}$

#### Experimental results: noise rates

#### Estimated noise rates are generally reliable



## Conclusion

## Learning from corrupted binary labels

Monotone relationship  $\bar{\eta}(x) = \phi_{\alpha,\beta,\pi}(\eta(x))$  facilitates:



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#### Future work - I

Better noise estimators?

• c.f. (Elkan and Noto, 2008) when D separable

More general noise estimators?

- e.g. learning from partial labels, multi-class corruption, ...
- see formulation of (van Rooyen & Williamson, 2015)

#### Future work - II

Alternatives to neural network for class-probabilities?

- choice of being unhinged versus neurotic
- for linearly separable D, Isotron (Kalai and Sastry, 2009)

Fusion with "loss transfer" (Natarajan et al., 2013) approach

- better for misspecified models
- assumes noise rates known

#### Future work - III

Applications:



### Thanks!